BRAIDED TENSOR CATEGORIES AND BRAIDED EXTENSIONS

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Abstract: This is a report on a joint work with Alexei Davydov.

Let C be a braided tensor category. A module braiding on a C-module category is an additional symmetry compatible with the braiding of C and giving rise to representations of pure braid groups. We show that braided C-module categories form a braided monoidal 2-category equivalent to the 2-center of the monoidial 2-category of C-module categories. In particular, invertible C-module categories form a braided categorical 2-group $\operatorname{Pic}_{\operatorname{br}}(C)$, the braided Picard group of C. We use this group to classify braided extensions of C graded by an Abelian group A. Namely, such extensions correspond to braided monoidal 2-functors from A to $\operatorname{Pic}_{\operatorname{br}}(C)$. These can be understood as usual braided monoidal functors such that a certain obstruction in the Eilenberg-MacLane abelian cohomology group $H^4_{\operatorname{ab}}(A, k^*)$ vanishes. We compute the group $\operatorname{Pic}_{\operatorname{br}}(C)$ in several examples and describe the corresponding extensions.