

# PARTIAL CO-REPRESENTATIONS OF HOPF ALGEBRAS

FELIPE LOPES CASTRO

Abstract: Partial actions arose in operator algebra study by Exel in [3] and it was extended to a purely algebraic context by Dockuchaev and Exel in [4]. The study of such a new structure has been developed in several directions, as the partial representation of partial group actions by Dockuchaev, Exel and Piccione in [5]. In [2], Caenepeel and Janseen developed the notion of partial actions of Hopf algebras on algebras, unifying partial actions of groups and actions of Hopf algebras. This new concept generates a rich theory, triggering the study of several aspects of this new structure.

In [1], Alves, Batista and Verduynselaer developed the concept of partial representation of Hopf algebras, retrieving properties from partial representation of groups to this new setting. In that work, the authors defined a partial representation of a Hopf algebra  $H$  on an algebra  $A$  as a linear transformation  $\pi: H \rightarrow A$  satisfying appropriated conditions. The authors defined the notion of partial  $H$ -module via partial representations of  $H$  on the endomorphism algebra  $\text{End}(A)$ , they constructed the category of partial  $H$ -modules, they constructed a universal algebra ( $H_{\text{par}}$ ) on which every partial representation is factorized and they showed  $H_{\text{par}}$  has a structure of Hopf algebroid.

The aim of this work is to present the study in development about the partial co-representations of coalgebras on Hopf algebras, exposing the properties and examples developed. A partial co-representation of a coalgebra  $C$  on a Hopf algebra  $H$  is defined as a linear transformation  $\omega: C \rightarrow H$  satisfying several conditions, duals of those presented in [1]. The initial examples are got from well-known partial structures, so examples of partial co-representation are obtained from partial representations and from partial coactions on coalgebras. In this work, we develop the notion of partial  $H$ -comodules, we construct a universal coalgebra  $H^{\text{par}}$  that factorizes every partial co-representation by coalgebra morphisms, we show that the category of partial comodules is isomorphic to the category of  $H^{\text{par}}$ -comodules and show that it has a structure of Hopf coalgebroid.

---

WORK IN COLLABORATION WITH MARCELO ALVES, ELIEZER BATISTA, GLAUBER QUADROS AND JOOST VERCRUYSSSE

## REFERENCES

- [1] M. Alves, E. Batista, J. Vercruyse. *Partial representations of Hopf algebras*, Journal of Algebra 426 (2015) 137-187
- [2] S. Caenepeel, K. Janssen. *Partial (co)actions of Hopf algebras and partial HopfGalois theory*, Comm. Algebra 36 (2008) 29232946.
- [3] R. Exel, *Circle actions on  $C^*$ -algebras, partial automorphisms and generalized Pimsner-Voiculescu exact sequences*, J. Funct. Anal. 122 (1994) 361401.
- [4] M. Dokuchaev, R. Exel. *Associativity of Crossed Products by Partial Actions: Enveloping Actions and Partial Representations*, Trans. Amer. Math. Soc. 357 (5) (2005) 1931-1952.
- [5] M. Dokuchaev, R. Exel, P. Piccione. *Partial Representations and Partial Group Algebras*, J. Algebra 226 (2000) 505-532.

DEPARTMENT OF MATHEMATICS, UNIVERSIDADE FEDERAL DE SANTA CATARINA, BRAZIL  
E-mail address: [f.castro@ufsc.br](mailto:f.castro@ufsc.br)  
URL: [www.mtm.ufsc.br/~fcastro](http://www.mtm.ufsc.br/~fcastro)