# HIGHER REPRESENTATION THEORY OF QUANTUM $\mathfrak{sl}(2)$

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Abstract: Representation theory is a fundamental tool in algebra. By turning a group into endomorphisms of vector spaces (i.e. matrices), one can understand a great deal about its structure. The idea behind higher representation theory is to not act on a vector space, but instead on a category, with the hope that the extra structure provided by the natural transformations will give us something more. When such an action comes from categorification (i.e. the lifting of a classical representation to the world of categories), it is closely related to the existence of nice basis in the underlying representation. This has already been proven useful to show the positivity of a basis in the Hecke algebra (Kazhdan-Lustig conjecture). Another motivation for studying such actions is given by low-dimensional topology: a lot of polynomial invariants for knots and 3-manifolds can be constructed using the representation theory of quantum groups (i.e. q-deformations of Lie algebras). Hence, it becomes natural to study their corresponding higher representation theory, and see if one can obtain homology theories from them. A common example is the Jones polynomial, which can be constructed using quantum sl(2), and the corresponding higher version turns out to be the celebrated Khovanov homology.

In this mini-lectures, I propose to explore a part of the higher representation theory of (quantum) $\mathfrak{sl}(2)$. It was first developed by Frenkel-Khovanov-Stroppel and independently Chuang-Rouquier with the categorification of the irreducible finite dimensional modules. We will then extend their work in order to categorify Verma modules, as done in a joint work with Pedro Vaz.

Preliminary plan:
**Lecture 1**: I'll start by explaining a bit more precisely what we mean by higher representation theory and categorification. I'll sktech some examples of categorification and give a very broad definition of categorical action. Then, I'll recall the basics of the classical representation theory of $\mathfrak{sl}(2)$. Finally, we will see how to construct a higher version of irreducible finite dimensional modules of $\mathfrak{sl}(2)$, using categories of modules over the cohomology of finite Grassmannian manifolds. For this, we will follow mainly the work of Frenkel-Khovanov-Stroppel.

**Lecture 2**: We will extend the construction of Lecture 1 by taking infinite Grassmannians and adding a bit of extra structure. Using tools from homological algebra and ideas from link homology, we will see how to use them in order to construct a higher version of Verma modules in the form of dg-structures.
**Lecture 3**: Hiding inside the higher structure of the finite dimensional modules is the nilHecke algebra. This algebra also comes naturally from the study of symmetric group actions on polynomial rings. Similarly, a dg-version of it sits inside the categorified Verma modules. We will discuss how these can also be used to construct equivalent categorifications as in Lecture 1 and 2. Finally, if time permits, we'll discuss some generalizations of it.