PROPERTY (T) FOR MONOIDAL CATEGORIES AND DISCRETE QUANTUM GROUPS

MATTHIAS VALVEKENS

Abstract:

Property (T) is a representation-theoretic rigidity property introduced by Kazhdan in the 60s to prove that certain lattices in higher-rank Lie groups are finitely generated. Roughly speaking, a locally compact group G has property (T) if a representation of G with almost invariant vectors must actually admit invariant vectors. Obvious examples include compact groups (in particular finite ones). Even in the class of discrete groups, nontrivial concrete examples are not that easy to come by; the most well-known family is probably $SL_(\mathbb{Z})$, $n \geq 3$, which arises directly from Kazhdans pioneering work. Note that this by no means implies that property (T) discrete groups are rare; A. uk showed in [**?**] that most hyperbolic groups have property (T), in some sense.

Over the years, the study of property (T) has attracted interest from multiple angles. In the late 90s, S. Popa introduced a version of property (T) for standard invariants of subfactors [**?**] – this can also be understood as a formulation for monoidal categories. Later, in 2008, P. Fima formulated property (T) for discrete quantum groups [**?**]. However, for a long time there were no known examples of either kind that were not explicitly constructed from discrete groups with property (T).

Based on a result of Y. Arano [**?**], S. Popa and S. Vaes showed in [**?**] that the representation categories of the compact quantum groups $SU_q(n)$ have property (T) for $n \geq 3$ and $q \neq 1$, thus providing the first examples of monoidal categories with property (T) that were not obtained from discrete groups.

In [**?**], we proved a monoidal category version of A. uks spectral criterion for property (T). Using the combinatorics of triangle presentations as the input, this led to a new family of examples of monoidal categories with property (T). These categories moreover give rise to discrete quantum groups with property (T) that cannot be built from property (T) discrete groups in any obvious way.

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