VARIABLE BANDWIDTH VIA WILSON BASES

BEATRICE ANDREOLLI

Abstract. Bandlimited functions are functions whose Fourier transform has compact support, which means that if the bandwidth of a function f is $\Omega > 0$, then Ω is the maximal frequency contributing to f. In contrast to bandlimited functions, which are entire functions of time and represent infinite signals, real-world signals are time-limited and therefore require a description that accounts for this fact. As a result, the concept of variable bandwidth arises naturally in signal processing and it is particularly intuitive when considering music, where the highest frequency (the note) varies with time. In this talk, we introduce a new space of variable bandwidth which is based on the frequency truncation of a Wilson expansion. Wilson bases, introduced in 1991 by Daubechies, Jaffard, and Journé [1], are orthonormal bases for $L^2(\mathbb{R})$ with a clear localization both in time and frequency. For these spaces, we study sufficient conditions and necessary density conditions for sampling. Moreover, by analyzing some numerical experiments, we motivate why these new spaces could be useful for the reconstruction of particular classes of functions.
The presentation is based on a joint work with Karlheinz Gröchenig [2].

References

[1] I. Daubechies, S. Jaffard, and J.-L. Journé. A simple Wilson orthonormal basis with exponential decay. *SIAM J. Math. Anal.*, 22(2):554–573, 1991. doi: 10.1137/0522035.

[2] B. Andreolli and K. Gröchenig. Variable bandwidth via Wilson bases. *Appl. Comput. Harmon. Anal.*, 71:101641, 2024. doi: 10.1016/j.acha.2024.101641.

Faculty of Mathematics, University of Vienna, Oskar-Morgenstern-Platz 1, 1090 Vienna, Austria

Email address: beatrice.andreolli@univie.ac.at