A Tour in Wavelet Phase Retrieval

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Wavelet phase retrieval consists of the inverse problem of reconstructing a square-integrable function f from its *scalogram*, that is from the absolute value of its *wavelet transform*

$$\mathcal{W}_\phi f(b,a) = a^{-\frac{1}{2}} \int_{\mathbb{R}} f(x) \overline{\phi\left(\frac{x-b}{a}\right)} \, \mathrm{d}x, \qquad b \in \mathbb{R},\ a \in \mathbb{R}_+.$$

The wavelet transform emerged from the research activities aimed to develop new analysis and processing tools to enhance signal theory, and has proved to be extremely efficient in various applications such as denoising and compression. However, there is still limited knowledge of the problem of reconstructing a function from the absolute value of its wavelet transform. More precisely, wavelet phase retrieval aims to determine for which *analyzing wavelets* ϕ and which choices of $\Lambda \subseteq \mathbb{R} \times \mathbb{R}_+$ as well as $\mathcal{M} \subseteq L^2(\mathbb{R})$ the forward operator

$$F_\phi : \mathcal{M}/\sim \to [0, +\infty)^\Lambda, \qquad F_\phi f(b,a) = |\mathcal{W}_\phi f(b,a)|, \quad (b,a) \in \Lambda,$$

is injective, where $f \sim g$ if and only if $f = \mathrm{e}^{i\alpha} g$ for some $\alpha \in \mathbb{R}$. In this talk, we present old and new results on this question and conclude by discussing some open problems.