Spaces of functions with nearly optimal time-frequency decay

Jasson Vindas

Let f be a one variable function such that both f and its Fourier transform $\widehat{f}$ satisfy the decay estimates

$$|f(x)| \leq Ce^{-rx^2} \quad \text{and} \quad |\widehat{f}(\xi)| \leq Ce^{-r\xi^2} \qquad \text{for some } r > 0. \tag{1}$$

According to Hardy's uncertainty principle, we must have $0 < r \leq 1/2$ if (1) holds for some non-identically zero f. Furthermore, in the extreme case $r = 1/2$, the only functions satisfying (1) are the multiple contants of the Gaussian $e^{-\frac{1}{2}x^2}$. In this talk we will study the Fréchet space of all those functions such that (1) holds for every $0 < r < 1/2$, which we call the space of functions with nearly optimal time-frequency decay. We shall present several descriptions of it in terms of the short-time Fourier transform and decay bounds for Hermite coefficients of its elements. We will also discuss counterparts of these characterizations for a broad class of subspaces defined via weighted inequalities refining (1). The main ingredients in our considerations are the Bargmann transform and some (weighted) forms of the Phragmén-Lindelöf principle on sectors. The talk is based on collaborative work with Lenny Neyt and Joachim Toft.