PROJECTIVE GEOMETRIC CODES

An Investigation into Small Weight Code Words

Sam Adriaensen – joint work with Lins Denaux, Leo Storme (UGent), Zsuzsa Weiner (Eötvös Lórand)

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VRIJE UNIVERSITEIT BRUSSEL

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For $c \in \mathbb{F}_p^{G_0(n,q)}$ we define

- $\text{supp}(c) = \{P \in G_0(n,q) \,||\, c(P) \neq 0\}$.
- $\text{wt}(c) = |\text{supp}(c)|$.

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To prove

Small weight code word are linear combinations of only a few k-spaces.

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Take $p \geqslant 5$ prime. Code words $c \in \mathcal{C}_1(2, p)$ with $\mathrm{wt}(c) < 3p - 3$ are lin. comb. of (at most) two lines.

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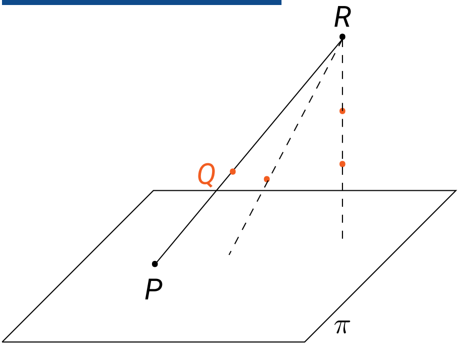
Arguments as in Lins' talk give us this:

Theorem

Take $p \geqslant 7$ prime. Code words $c \in \mathcal{C}_k(k+1, p)$ with weight below roughly $2.5p^k$ are lin. comb. of (at most) two k-spaces.

SECOND INDUCTION STEP

THE PROJECTION MAP



We go from results of $\mathcal{C}_k(k+1,p)$ to results of $\mathcal{C}_k(n,p)$. We use the following projection map.

$$\mathrm{proj}_{R,\pi}(c) : P \mapsto \sum_{Q \in RP} c(Q).$$

Then $\mathrm{proj}_{R,\pi}(c) \in \mathcal{C}_k(n-1,p)$.

Original idea: M. Lavrauw, L. Storme, G. Van de Voorde

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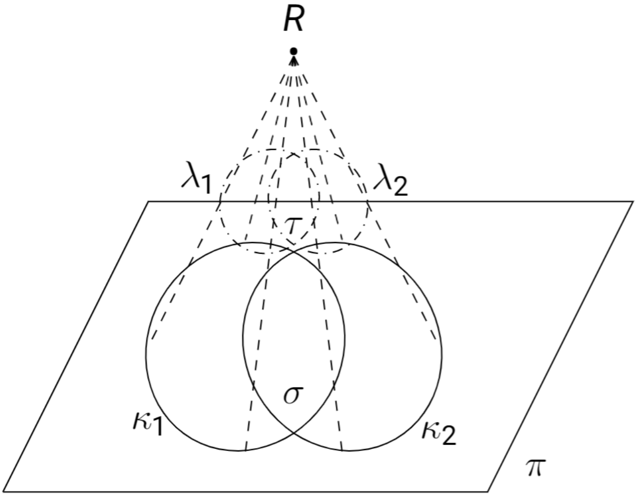

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Using field reduction

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Assume that $c \in \mathcal{H}_1(2, q)$, and take $P \in \text{supp}(c)$. All points $Q_i \in \text{supp}(c) \setminus \{P\}$, s. t. $|PQ_i \cap \text{supp}(c)| = 2$ are collinear.

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- This yields several collinear points of $\text{supp}(c)$.
- We can keep repeating this argument until we find a line $l \subseteq \text{supp}(c)$. All points of l have the same coefficient in c.
- It is not hard to go to a contradiction.

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Using the previous induction tools we obtain:

Theorem

The minimum weight of $\mathcal{H}_k(n, q)$ equals $2q^k$.

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Problem again: $\perp$ reverses inclusion, so we can't use field reduction.

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Bagchi & Inamdar introduced a pull-back contruction, to go from $\mathcal{C}_{0,k}(n,q)^\perp$ to $\mathcal{C}_{j,k+j}(n+j,q)^\perp$. It means that all elements of $\mathrm{supp}(c)$ go through a fixed $(j-1)$-space.

Conjecture (B. Bagchi, S. P. Inamdar)

All minimum weight code words of $\mathcal{C}_{j,k}(n,q)^\perp$ are pull-backs if q is prime.

They proved for $j = k-1$.

Theorem

All minimum weight code words of $\mathcal{C}_{j,k}(n,q)^\perp$ are pull-backs.

The problem reduces to $j = 0$.

THE DUAL CODE

FURTHER REDUCTION

Together with previous results (M. Lavrauw, L. Storme, G. Van de Voorde), we can reduce the minimum weight problem to codes of the form $\mathcal{C}_{0,1}(n, q)^{\perp}$.

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The minimum weight of $\mathcal{C}_{0,1}(n,q)^{\perp}$ is

- known and characterized for q prime.
- known for q even.

POSSIBILITIES FOR FURTHER RESEARCH

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- Determine the dimension in general. This is only known for $j = 0$, and, by duality, $k = n - 1$.
- Examine some generalizations of these codes. I am currently looking at the code generated by j-spaces in a k-space through an i-space.

Thank you for your attention!