



BELGIAN MATHEMATICAL SOCIETY

Comité National de Mathématique CNM

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Nationaal Comite voor Wiskunde



BMS-NCM NEWS: the Newsletter of the Belgian Mathematical Society and the National Committee for Mathematics

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BMS-NCM NEWS

No 94, September 15, 2013



Letter from the editor

Welcome to this September edition of our Newsletter.
Have a nice semester!

Regards, Françoise

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1 News from the BMS & NCM

1.1 PhD-Day

The 4th

PhD-day of the BMS-NCM

took place in Brussels (Academy) on

Monday September 09, 2013.

Thanks to all participants. The “winner posters” are at the end of the Newsletter.

1.2 Bulletin of the BMS - electronic version

We remind you that it is possible to convert your paper subscription to the Bulletin of the BMS into **the electronic version of the Bulletin. If you are interested, please contact Philippe Cara by e-mail** (pcara@vub.ac.be with bms@ulb.ac.be in cc) for details.

You will receive a special “subscriber code” with which you can register for the Bulletin of the Belgian Mathematical Society at Project Euclid (<http://projecteuclid.org>).

2 Meetings, Conferences, Lectures

2.1 October 2013

Nord Pas de Calais/Belgium congress of mathematics
Valenciennes and Mons, October 28-31, 2013

Event: Nord Pas de Calais/Belgium congress of mathematics

Dates: Oct. 28-31, 2013

Location: Valenciennes and Mons

Email: ncpbe@umons.ac.be

Web site: <http://math.umons.ac.be/ncpbe13/>

Summary: This congress, organized by the Departments of Mathematics of Belgium and of the North of France, will consist of eight plenary lectures and ten parallel sessions. It will cover many research topics common to the Departments, ranging from functional analysis to game theory, through PDE, numerical methods, algebra and differential geometry.

2.2 November 2013

Analysis Meeting
on the occasion of the 60th birthday of our colleague and friend
Gilles Godefroy
Mons, November 4-5, 2013

See the website at the address www.afo.ulg.ac.be/fb/meeting/gilles2013 and also the poster at the end of this Newsletter.

For more information:

Catherine Finet, UMONS (catherine.finet@umons.ac.be)

Françoise Bastin, ULg (F.Bastin@ulg.ac.be)

Théorie des modèles des corps
Mons, Lundi 18 novembre 2013

Le lundi 18 novembre 2013 à l'UMONS aura lieu une journée sur le thème “THEORIE DES MODELES DES CORPS”, dans le cadre de l'Ecole Doctorale Thématische Mathématique.

Informations pratiques:

- Lieu : UMONS, Campus de la plaine; Bâtiment *Le Pentagone*, salle 3E20
- L'accueil débutera à 9h45
- Pour l'inscription à la journée et au lunch, veuillez contacter l'un des organisateurs : quentin.brouette@umons.ac.be, francoise.point@umons.ac.be, christian.michaux@umons.ac.be

Les conférenciers prévus sont :

- Salma Kulhmann (Université de Konstanz, Allemagne),
- Mickael Matuzinski (Université de Bordeaux, France),
- Nathanael Mariaule (Seconda Universita di Napoli, Italie),
- Samaria Montenegro (Université Paris 7, France),
- Franziska Jahnke (Université de Münster, Allemagne).

3 PhD theses

Techniques from monoidal category applied to generalized Lie algebras and various types dualities

Isar Goyvaerts

Date: VUB, August 23, 2013

Promotors: S. Caenepeel and J. Vercruyse

Summary

Lie algebras play a prominent role in a large number of different branches of mathematics. In the past decades, various generalizations -in different directions- of this algebraic structure have appeared in literature. Motivated by the way the field of Hopf algebra theory benefited from the interaction with the theory of monoidal categories on the one hand, and the strong relationship between Hopf algebras and Lie algebras on the other hand, the natural question arose whether it is possible to study Lie algebras within the framework of monoidal categories, and whether Lie theory could also benefit from this point of view. Some of these connections between the above-mentioned theories can be described by means of certain duality theorems. The concept of duality -in a broad sense- and certain types of autodual objects in particular, are of considerable interest in the monoidal categorical context.

The main goal of this work is, roughly speaking, twofold. On the one hand, we wish to establish different types of dualities for generalized Lie (and Hopf) algebras; as Lie (and Hopf) algebras allow for a study in general additive, symmetric monoidal categories, one can expect the afore-mentioned duality results to be extended to this more abstract setting, allowing to formulate duality theorems for those generalized structures which are not covered by the classical approach. On the other hand, and of a slightly different nature, we propose a study of a particular type of autodual objects in certain braided monoidal categories, which are in close connection to quadratic modules. The chief aim here is to provide a tool which can be used to understand some aspects of the structure of categories of representations of finite groups.

Robust Alternatives to Least Squares for Sparse Estimation with Applications

Vahid Nassiri

Date: September 4, 2013

Thesis Advisors: Philippe Cara and Ignace Loris

Summary

In this work, sparse robust estimation of the parameters of a linear regression model is considered. Consider y , the response variable that we study using a set of k so-called explanatory variables $X = (x_1, x_2, \dots, x_k) \in \mathbb{R}^{n \times k}$ which are linearly related to the variable y :

$$y = X\beta + \epsilon,$$

where β is a vector containing k parameters, and ϵ is the error of the model. A sparse estimation of β can be obtained using a penalized model. Such a penalized model consists of three main aspects: the loss function T , the penalty function g and the solution method to obtain the estimated parameters $\hat{\beta}$ by solving the optimization problem:

$$\hat{\beta} = \operatorname{argmin}_{\beta} T(y - X\beta) + \lambda g(\beta),$$

($\lambda \geq 0$ is the penalty parameter). We are going to study each of these three aspects throughout this thesis.

For the loss function part, we will introduce some robust alternatives to the least squares loss function. These (possibly asymmetric) loss functions are convex and differentiable. The first loss function is based on a novel class of probability distributions: the so-called connected double truncated gamma distribution. We show that using this class as the error distribution of a linear model leads to a generalized quantile regression model that combines desirable properties of both least-squares and quantile regression methods: robustness to outliers and differentiable loss function. Furthermore, the possibility of using a skew-symmetric class of distributions as error distribution in a linear model is investigated. Selecting appropriate kernels in this class, would provide us with a robust alternative to least-squares which is convex and differentiable.

For the penalty part, we consider mixed norm penalties of types $g(\beta) = \|\beta\|_{1,\infty}$ and $\|\beta\|_{1,2}$, which allow the researcher to impose structured sparsity on groups of variables instead of the individual ones. Three main

applications of such penalties will also be considered. We will use it to include a nominal variable (with more than 2 levels) in a penalized quantile regression model. It will also be used for simultaneous variable selection in a multivariate quantile regression model. In addition, a novel application of such structured sparsity will be introduced in the field of impact force identification.

For the solution method, we will consider two classes of efficient algorithms: iterative algorithms and non-iterative algorithms. By efficiency we mean speed of convergence (in the iterative case). But for non-iterative algorithms, efficiency means the ability to give the solution $\hat{\beta}(\lambda)$ for a range of possible values of λ at once. A novel algorithm will be derived to solve a structured sparse quantile regression with a $\ell_{1,\infty}$ -norm penalty. The algorithm gives the piece-wise linear solution path for $\hat{\beta}(\lambda)$ in a finite number of steps.

Existence and non-existence of hypercyclic subspaces

Quentin Menet, Université de Mons

Date: 16:00, November 15, 2013

Local: Marie Curie, Grands Amphithéâtres, Université de Mons

Thesis Advisor: K.-G. Grosse-Erdmann (UMONS)

Jury: G. Godefroy (IMJ, France) (President), T. Brihaye (UMONS) (Secretary), C. Finet (UMONS), S. Grivaux (Lille 1, France), E. Matheron (Université d'Artois, France) and A. Peris (UPV, Spain)

Summary

Linear dynamics studies the properties of orbits of operators on Banach or Fréchet spaces. A key notion of linear dynamics is the notion of hypercyclic operators. An operator T on a Fréchet space X is said to be hypercyclic if there is a vector x in X (also called hypercyclic) whose the orbit under T is dense. In this thesis, we focus on the notion of hypercyclic subspaces. We say that an operator T possesses a hypercyclic subspace if there exists an infinite-dimensional closed subspace in which every non-zero vector is hypercyclic. In 2000, a characterization of operators with hypercyclic subspaces was obtained by González, León and Montes in the case of complex Banach spaces by using spectral theory. However, so far no characterization of operators with hypercyclic subspaces on Fréchet spaces is known. The investigation of the existence and the non-existence of hypercyclic subspaces for operators on Fréchet spaces is the main goal of this thesis.

In a first time, we characterize weighted shifts with hypercyclic subspaces in certain Fréchet sequence spaces such as the space of entire functions. This result generalizes the existence of hypercyclic subspaces obtained by Shkarin in 2010 for the derivative operator by determining which weighted shifts on the space of entire functions possess a hypercyclic subspace. In the case of Fréchet spaces without continuous norm, we remark that there exist two types of closed infinite-dimensional subspaces and thus two types of hypercyclic subspaces. We develop criteria for the existence and the non-existence of hypercyclic subspaces of each of these two types. These results allow us to answer positively a question posed by Bès and Conejero by proving the existence of operators with hypercyclic subspaces on each separable infinite-dimensional Fréchet space. Finally, while so far no characterization of the existence of hypercyclic subspaces in the case of Fréchet spaces is known, we succeed to obtain a characterization of sequences of operators possessing hereditarily hypercyclic subspaces.

In a second time, we investigate the spaceability of the set of restricted universal series, the notion of hypercyclicity for a subset and the existence and the non-existence of frequently hypercyclic subspaces. In particular, we show that, for any Fréchet space non-isomorphic to ω , the existence of a restricted universal series implies the spaceability of the set of restricted universal series and we exhibit a frequently hypercyclic weighted shift on l^p with a hypercyclic subspace and without a frequently hypercyclic subspace. This latter example allows us to answer positively an open problem posed by Bonilla and Grosse-Erdmann.

4 From EMS

4.1 Newsletter

The September issue of the Newsletter of the EMS is on line: <http://www.ems-ph.org/journals/journal.php?jrn=news>

4.2 Call for nominations or proposals

Call for nominations or proposals of speakers and scientific events in 2014

The EMS has published the call for nominations or proposals of speakers and scientific events in 2014. Information on the calls and on the submission procedure is given at

<http://www.euro-math-soc.eu/node/3266>

Deadlines for distinguished speakers: December 30, 2013.

5 Miscellaneous

5.1 From KUL

Dear colleagues,

The University of Leuven (Belgium) is advertising a **position in geometry**. The position is open to researchers in any area of geometry or its interactions with other domains, e.g. geometric group theory or analysis on manifolds.

The job ad can be found at <http://icts.kuleuven.be/apps/jobsite/vacatures/52484362?lang=en>

Depending on qualifications and track record, the appointment can be in any of the grades of the senior academic staff (from tenure track assistant professor to full professor).

The application deadline is September 30, 2013.

5.2 From University of Antwerp

The Department of Mathematics of the University of Antwerp is looking for a **Postdoc position in Communication hiding Krylov methods**.

Job description

We are hiring a postdoctoral researcher focusing on the development of numerical algorithms and software for future exascale HPC systems. Specific research topics are communication hiding and avoiding techniques in Krylov subspace solvers. The postdoc will join a European project on exascale software that involves researchers from INRIA France, USI Switzerland and VSB Czech republic. The work will also be done in close collaboration with the ExaScience Lab Flanders (www.exascience.com; <http://www.exascience.com/>), a research partnership between Intel, Imec research center and five Belgian (Flemish) universities KU Leuven, U Gent, U Hasselt, VUB and U Antwerpen.

Profile

The candidate is expected to have a demonstrated track record in parallel scientific computing and the numerical analysis of Krylov subspace methods. He/she must be able to work efficiently with other researchers in this collaborative research environment. Fluency in English is required.

Offer

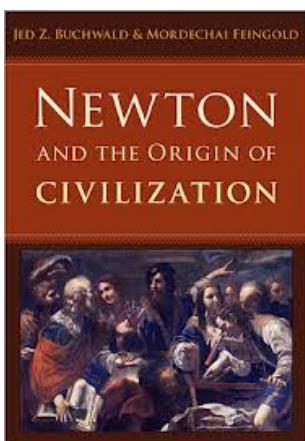
The successful candidate will be offered a position for one year with a possible extension to a total period of three years.

Interested?

For applications and informal inquiries, please contact Wim.Vanroose@ua.ac.be; <mailto:Wim.Vanroose@ua.ac.be>; or Karl.Meerbergen@cs.kuleuven.be; <mailto:Karl.Meerbergen@cs.kuleuven.be>.

6 History, maths and art, fiction, jokes, quotations ...

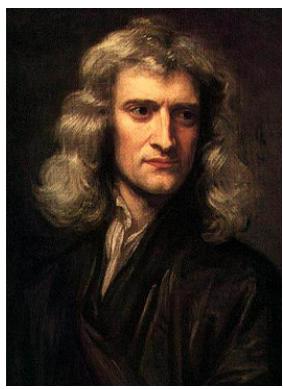
Newton and the origin of civilisation *Jed Z. Buchwald and Mordechai Feingold* Princeton University Press, 2013 (xvi+528 p.), hard cover, ISBN 978-069-115478-7, £ 34.95



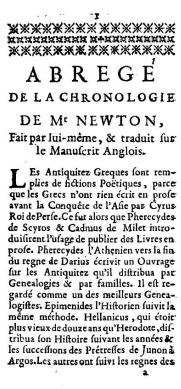
We all know Isaac Newton (1642-1727) as an all-round scientist, being a mathematician, engineer, physicist, astronomer, philosopher, alchemist, and theologian. His name is connected to many mathematical theorems, laws and algorithms. His *Philosophiae Naturalis Principia Mathematica* (1687) laid the foundations of classical mechanics and is considered his masterpiece. In his *Opticks* (1704) he showed how white light can be decomposed into different colours with a prism and rebundled into white light with a lens. He was not only a developer of theories, he was also good at doing experiments and designed a telescope named after him. And this list can be continued for a while.

Mathematicians probably do not know so much about one of his lesser successes, namely his work on chronology. Although the principles and skills that he applied in this study originate from his early career, he got only fully involved in the time-line of early history at a later stage in life, but still, on and off, he has put effort in the topic during some 50 year. An *Abstract* of his notes circulated before his death. It was translated into French and published together with (anonymous) comments by Etienne Souciet as *Abbrégé de la chronologie de M. le Chevalier Isaac Newton, fait par lui-même* (1725). The full notes were only published posthumously as *The Chronology of Ancient Kingdoms Amended* (1728). It starts with a letter to the Queen of England by John Conduitt (member of parliament and married to Newton's niece) followed by *A short chronicle* which is nothing but a list of dates and events ranging from 1125 BC to 331 BC. The rest consists of six chapters

- Chap. I. Of the Chronology of the First Ages of the Greeks.
- Chap. II. Of the Empire of Egypt.
- Chap. III. Of the Assyrian Empire.
- Chap. IV. Of the two Contemporary Empires of the Babylonians and Medes.
- Chap. V. A Description of the Temple of Solomon.
- Chap. VI. Of the Empire of the Persians.

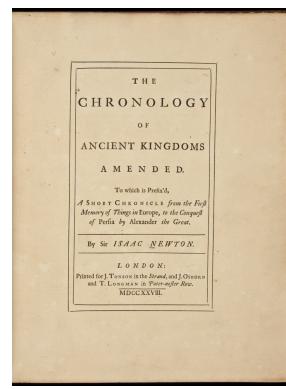


I. Newton (ca 1689)



Abrégé
DE LA CHRONOLOGIE
DE M^e NEWTON,
Fait par lui-même, & traduit sur
le Manuscrit Anglois.

Les Antiquités Grecques sont tem-
ples de fictions Poétiques, parce
que les Grecs n'ont rien écrit en prose
avant la Conquête de l'Afie pas Cyrus
qui a été fait au commencement des
Scyros & Cadmus de Milet immo-
diument l'usage de publier des Livres en
profe. Pherecydes l'Athenien vers la fin
du règne de Darius écrivit un Ouvrage
qui fut à Athènes et dans d'autres villes
Généalogies & par familles. Il est con-
sideré comme un des meilleurs Gene-
alogistes. Epiménides l'Histiorien suivit la
même méthode. Hellanique, pour écrire
plus tard une Histoire de l'Asie, a choisi
tribus ion Histoire suivant les années &
les succéssions des Prétendus de Jounos à
Argos. Les autres ont suivi les règnes des



Abrégé (1725)

Chronology (1728)



I. Newton (ca 1712)

In this book Newton revised the ancient history of the kingdoms mentioned in the chapters. He used all sources of ancient scriptures available, combined it with his knowledge of astronomy and tables and maps of observations to date the events in the past. The result was a contraction of history as it was commonly accepted in his days and it resulted in a vivid controversy among scientists pro and contra his findings. In their book, Buchwald and Feingold (B&F) scrutinize the archives to explain why, how, and when, Newton has come to his results. Newton has gone through all the literature available to him to find dates and references, but the authors of this book did as

well. It is as if they let the reader look inside the head of Newton. Every writing, letter, or public statement that has been made concerning this matter by Newton as well as by his opponents and his followers is analysed, explained, and continuously cited or quoted.

They start from the early beginnings to illustrate Newton's way of combining theory and experiments to arrive at a conclusion. In their first chapter they elaborate on the question of how far we can trust our senses. *Quaestiones quaedam philosophicae* is a set of notes by the young Newton where he contemplates on natural philosophy and the proper way to generate reliable knowledge. Every answer to a question should be subject to experiment, a motto that Newton has guided through his whole research career. As Newton reads the philosophers, B&F take the reader along from the hylomorphism of Aristotle, over scholastic cognition by Thomas Aquinas till the mechanical views of René Descartes. The astronomical observations by Newton, not only made him think about how to trust the sense of sight and the limitations of the eye, it also trained him as an experienced observer.



J. Hevelius



R. Hooke



C. Huygens



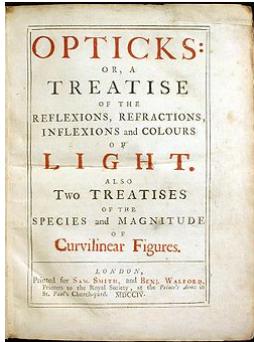
R. Boyle

The second chapter is about the controversy between Hevelius and Hooke. Johannes Hevelius (1611-1687) was an astronomer in Danzig who did his observations with “naked eye devices” while in Britain, Robert Hooke (1635-1703) was claiming that it was impossible to do observations properly if not by using telescopes with cross-hairs. Hooke uses members of the Royal Society to prove that human senses are untrustworthy, yet Hevelius’s numbers were more accurate than expected because he did several observations and then selected the one assumed to be most accurate. This resulted in a public discussion. Similarly Christiaan Huygens (1629-1695) was convinced of this uncertainty of observations even if done with his (inevitably imperfect) instruments (clocks, telescopes), while Robert Boyle (1627-1691) was convinced that experiments can produce reliable results. Newton also made several observations but his smart and in those days original idea was to take the average, and replace even the observation that was the most accurate with this average. This was unprecedented. P&G illustrate that although taking averages was applied in other publications, it was done for different purposes. They illustrate this idea in Newton’s case with his experiments about the diffraction of light, published in his *Opticks*. It also shows Newton’s way of thinking: he starts from an hypothesis (a model), then does experiments and let these, through computation, converge into a single number. This scientific approach he applied in all other situations as well.

In their chapter 3, B&F sketch erudition and chronology in 17th-century England. They show the evolution of chronology from Julius C. Scalinger (1484-1558), over Jean Bodin (1530-1596), François Béroalde de Verville (1556-1626) up to John Selden (1584-1654) and how technical chronology raised although eventually it lost importance in subsequent centuries. Of course the obvious clash between the study of laic documents and the infallibility of the Christian Bible was a constant source of severe discussion.

The next chapter describes Newton’s vision on idolatry and prophesies. For example, at some

point he predicted the end of the world to happen in 2694. He came to that conclusion because he



Newton's Opticks



J.C. Scalinger



J. Bodin



J. Selden

situated the fall of Jerusalem in AD 69, to which he added 1290 and 1335, numbers that he found in a prophecy of Daniel. He scribbled this calculation on the back of an envelope. His *Observations upon the Prophecies of Daniel, and the Apocalypse of St. John* were also published posthumously. However, do not start worrying yet, this date for the apocalypse is just one possibility. He did compute several other dates. Newton saw these prophecies as symbols that should be linked to events, to which he then aligned his astronomical computations. Concerning idolatry, Newton was convinced that the gods from mythology were in fact sublimations of kings, and hence that mythology actually described ancient history. He even linked Egyptian myths with the kingdoms of Noah and his sons. The latter populated the world after the deluge and this history was transformed into Greek mythology. He even morphed these events into the description found in Genesis. He did not believe that the earth was created from chaos.

In chapter 5, the topics are the calculations by Newton and others about the world population after the deluge and the discussion about the deluge being a local rather than a world-wide event. Some of the models were completely ridiculous, assigning high fertility to the Jews, resulting in populations before and after the deluge that are multiples of the current world population. This discussion about population dynamics led Newton to reduce the number of people living in ancient history which consequently forced him to compress ancient history, a conclusion that is described in the next chapter. In the book *Daniel* of the Old Testament, 4 kingdoms are mentioned: Babylon, Persia, Greece and Rome. So Newton was wondering what had happened to Egypt and Assyria. His view on the dynamics was that after the deluge, the population was sparse, but with growing population, towns were formed, these needed a judge or some authority, and these cities eventually formed kingdoms that were enlarged by conquest or merging. He claimed that it was only in the time of Moses that the first city-kingdoms emerged. Assyria's capital Ninive, two centuries after the deluge was still a small city. Egypt in Moses' time was a collection of smaller kingdoms. The historical events he places very late in history. Solomon's death was 980 BC, the Argonauts 42 years later. He disavowed that the four ages of men under the reign of Noah and his successors Ham, Chus, and Nimrod. He claimed these did not happen in succession, but in parallel. He identifies them with Egyptian pharaohs Amosis, Ammon, Sesostris and Orus, the first three corresponding to Saturn, Jupiter and Mars.

The three sources Newton used to come to this drastic abbreviation of history are described in Chapter 7: the *Persika* by Ctesias of Cnidus (5th century BC) for the Persian history, the *Aegyptiaca* by Manetho (3rd century BC) for the history of Egypt and the *Marmor Parum* (ca 264 BC) for the Greek. However words should be linked to observations. And so he did. Newton developed a way to date events by locating colures at certain positions in the zodiac. On the helical sphere on which the stars evolve around the earth, the colures are great circles (not on the figure) through the celestial poles which align with the earth poles (inclined axis). One goes through the equinoxes, and the

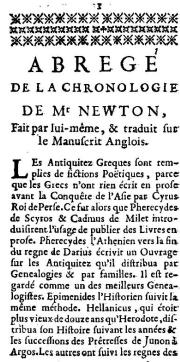
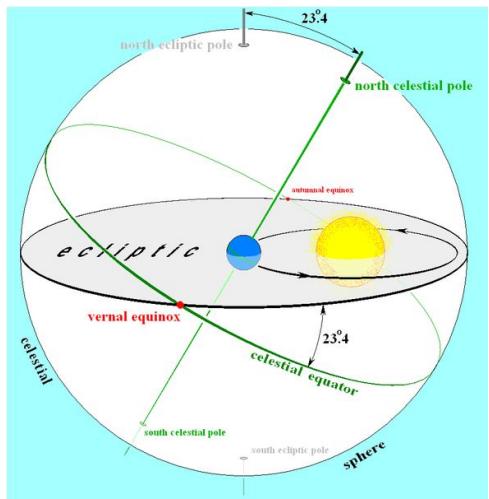
**OBSERVATIONS
UPON THE
PROPHECIES
OF
D A N I E L,
AND THE
A P O C A L Y P S E
O F
St. J O H N.**

*In Two PARTS.
By Sir ISAAC NEWTON.*

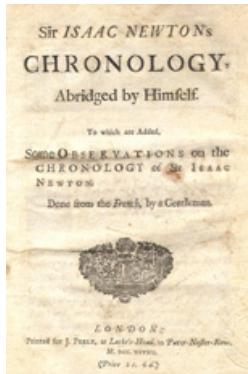
*LONDON,
Printed by J. DODS and T. BOWEN in Bartholomew-Court,
And Sold by J. DODS and T. BOWEN in the Strand,
W. TUCKE and M. MANSON in Fleet-Street,
Paul's-Church-Yard, J. CHURCHILL and T. LOUSET in Pall-Mall,
J. DODS and T. BOWEN in Cheapside, and T.
HATCHETT at the Royal Exchange, S. HARDING in St.
Paul's-Church-Yard, and T. PARKER in
Paternoster-Row, and J. BAINES in New-Bond-Street.
MDCCLXXXIII.*

other, orthogonal to it goes through the solstices. The zodiac is found along the celestial equator, orthogonal to the two colures. The ecliptic (horizontal circle) has an inclination of $23^{\circ}4$ with respect to the equator. The equator, the ecliptic and the equinoctial colure intersect in the vernal and autumnal equinoxes. The position of the equinoxes in the zodiac shifts over the ages because of precession of the earth. It is this shift that allowed Newton to transform astronomical observations in the scriptures to dates. Therefore he had to rely on images with precise constellations of the stars. These were provided by publications of Hevelius and of John Flamsteed (1646-1719).

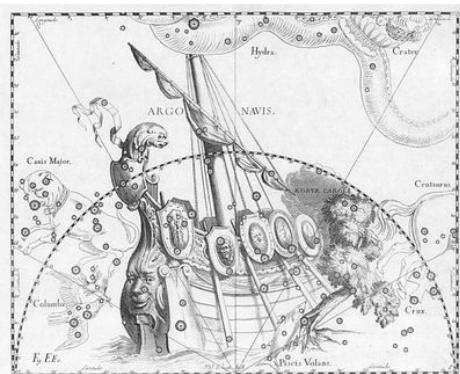
In the following 3 chapters, B&F discuss the publication of Newton's results on chronology and the flaming discussions it launched among scientists. First Newton made an abstract for Princes Caroline (1713-1757) who lent it to Abbé Conti (1677-1749) who was the intermediary in the Newton-Leibniz calculus controversy. The latter made a copy and that was translated to French and published in Paris as the *Abrégué* with comments anonymously added by Etienne Souciet (1713-1757), but without Newton's consent. This version of the story and variations of it are investigated by the authors. Other preliminary knowledge was shown by John Senex (1678-1740) who started producing globes on which Newton's colures appeared. With the posthumous publication of the full notes, and the translation of the *Abrégué* into English *Sir Isaac Newton's Chronology: Abridged by Himself. To which are Added, Some Observations on the Chronology of Sir Isaac Newton* (1728), the war over the topic broke loose.



Abrégé



Abridged



Argo Navis constellation
of stars by Hevelius



J. Flamsteed

William Whiston (1667-1752) was one of the opponents in England. He had read the original, but his publications were not very coherent and sometimes he wanted to use Newton's arguments for his own benefit, which brought him into contradictions for which he was attacked by other opponents of Newton's chronology. In France Nicolas Fréret (1688-1749) took over from Souciet and, although less of a mathematician, nevertheless contested Newton's calculations. Edmond Halley (1656-1742) and Louis Jouard de La Nauze (1696-1773) on the other hand joined the discussion in defense of Newton in England and France respectively. B&F describe the controversy in two full chapters and gradually move on to the next one in which is described how eventually the conflict and in general the interest in technical chronology as it was in those days faded away in the subsequent decades.

In several appendices B&F give a useful list of technical definitions, and conventions and they describe Newton's method of calculation and they comment on extracts from Newton's notes for the *Chronology* and the calculations in there, how he places the colures. For example appendix D explains how colures can be placed on the original star globe. First, a star is chosen that lies (almost) on the colure. Then the position of that star is looked up in Hevelius or in a catalog

originating from Flamsteed. These give the coordinates of the star in the zodiac in 1690. Taking the precession into account allows to find the original date. The computations are relatively simple for the solstitial colure, but for the equinoctial colure, if the star is not on the ecliptic, it is a bit more complex.



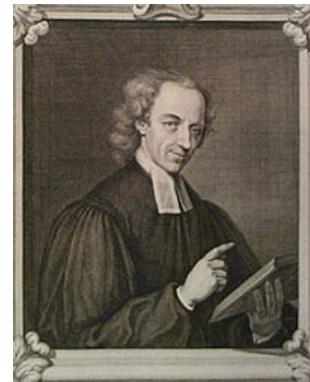
Princes Caroline



Senex globe



Abbé Conti



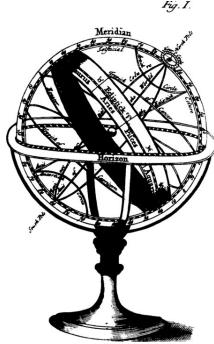
W. Whiston

Appendix E explains how Newton obtained two confirmations for his dates: one for Hesiod and one for Thales. He gives no hints on how he does it in the *Chronology*, but much can be learned from his notes which is again illustrated by B&F.

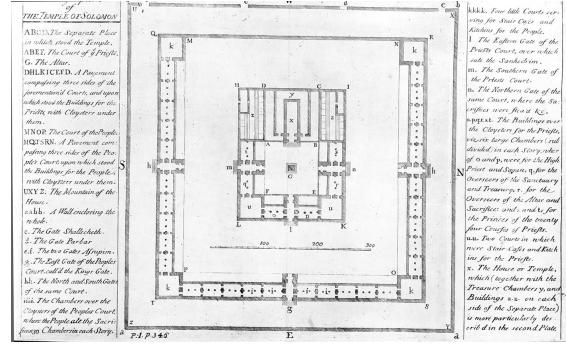
An extensive list of references and a dexterous index complete this extensive and in depth study.



Telescope designed
by Newton



Globe used by Newton



The temple of Solomon as it was published
in the *Chrononogy*

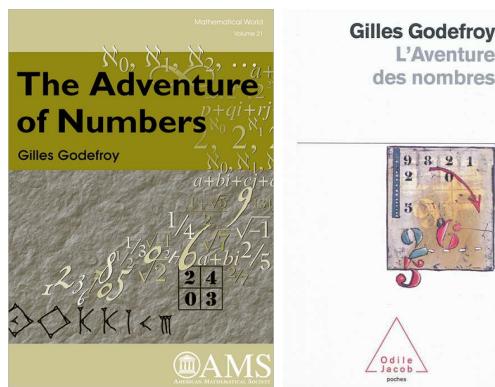
Many of the historical works by Newton can be found at the site of the Newton project (www.newtonproject.sussex.ac.uk), or at the project Gutenberg (www.gutenberg.org). Although not essential, it will certainly be helpful if the reader is familiar with the Old Testament, or possibly other old scriptures, with ancient history, and with Greek mythology. If you are not so familiar, you may need a lot of looking up in wikipedia or elsewhere about who-is-who and where-is-where in ancient history.

With this book Buchwald and Feingold have provided the specialists with an overwhelming source of information. But anyone interested in history, i.e., ancient history but also the history of the 17th century and the evolution of ideas in between, will love the overwhelming stream of detailed information. The reader might otherwise be interested in the biography of Newton or just the methods and approaches used by a genius. Although this is a long review, it mentions only a tiny bit of everything that has been discussed in this book. The style and vocabulary used is not the simplest, but nevertheless it reads fluently. The only problem may be that in a novel there are usually not more than a handful of main characters. Here the reader may have a hard time to keep all the players, their motives, and the changing versions of events and results apart.

Adhemar Bultheel

L'aventure des nombres *Gilles Godefroy* Éditions Odile Jacob , Paris, 1997, (237 p.), ISBN 978-2-7381-0422-9, and in pocket edition, 2011 (ix+234 p.), ISBN 978-2-7381-2551-4.

The adventure of numbers *Gilles Godefroy* Vol. 21 of Mathematical World, AMS, 2004 (ix+194 p.), soft cover, ISBN 978-0-8218-3304-9.

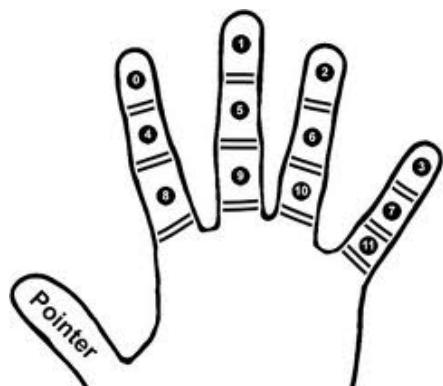


Although the original version dates back to 1997, and the English translation was published in 2004, the release of the French version in the series *Poches Odile Jacob* in 2011 is a good reason to take another look at this book.

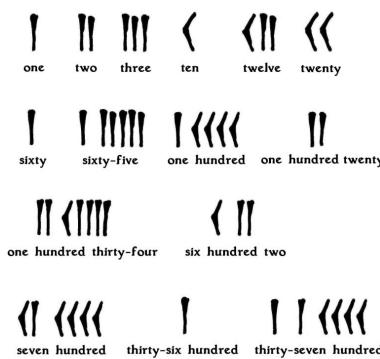
The title may suggest that this is one of these books popularizing mathematics for the layman. However, I do not think that a non-mathematician will reach the end of the book. Godefroy sketches the history of numbers and number systems, starting with the most primitive attempts of man to count on their fingers but it ends with all the major questions and challenges that mathematics

is faced with at the end of the twentieth century.

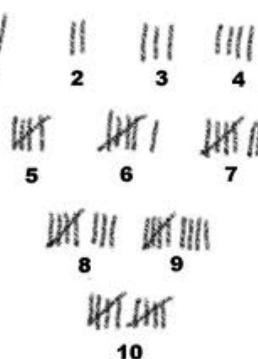
The start is easy and smooth and will be appreciated by the mathematician as well as the non-mathematician. Although humans usually have ten fingers (digits!), the decimal system seems to be the most natural, but also the binary system (one-two-many) is in our digital computer era a widespread habit. Nevertheless, we also live with several traces of the number system in base 12. In a sense, this is also natural if you count on your fingers. Use your thumb as a pointer at the twelve finger bones of the four other fingers. If you count the multiples of 12 with the fingers of your other hand, you end up with 60. In a sense 12 is more convenient for calculating since it has more divisors than 10.



counting to 12



Sumerian numbers



tallies

Counting is one thing, writing down numbers is another. With commerce flourishing in old Sumeria (about 3500 BC) there was a need to have a physical way to denote quantities that were bought or sold. The first numbers were written on clay, with little marks each representing one unit. This is what we still do when we keep scores writing tallies. Cutting notches in a stick for commercial reasons was common until relatively recently. The French word for notches is *tailles*, and this crossed to English as *tallies*. One cuts notches on tally sticks. The longest stick (stock) is kept by the lender and the other is taken by the borrower. And this gave rise to the word *stockholder*. But sticks with notches were not the only way of course. Also stones (small pebbles) were used (calculus is Latin for small pebble). Basically this is what is done on an abacus, an instrument still very much in use in China. Whether it are stones or notches, a systematic way had to be invented to group them in larger quantities. That required a major step towards a positional number system and brings us to the Sumerians and Babylonians. To write numbers Sumerians used the *nail* and the *wedge* as the only symbols. Their system gradually evolved into a positional

system in base 60, much like we denote our decimal numbers. But a positional system also needed an indication that some power of 60 is missing. So there is already a need for the number zero.



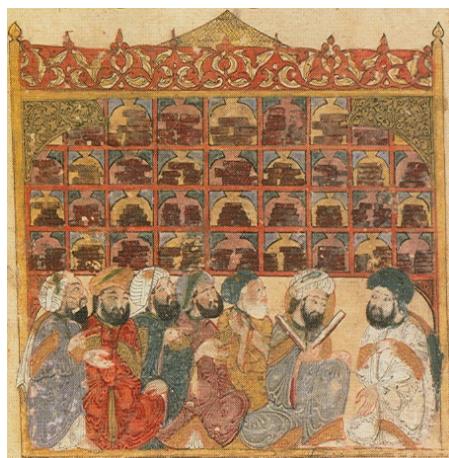
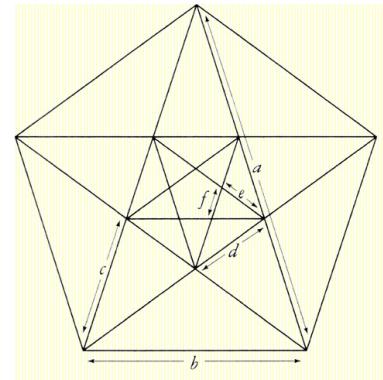
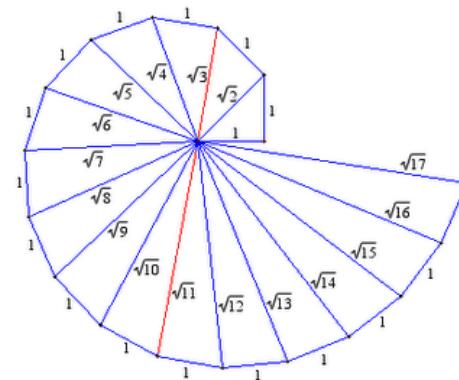
tally sticks



battle of Marathon

The Babylonian kingdom ended when it fell to the Persian emperor Cyrus the Great in 539 BC. The Greek were shielded away from the Persian expansionism when they won the battle of Marathon in 490 BC. It was in that Greece that the Pythagorean school that was founded around 600 BC came to full expansion. For them integers were the foundation of knowledge, leading to an atomistic view of space. All lengths were (finite) integer multiples of units (*nomads*). Taking the ratio of two lengths then gave rise to a rational number that was the ratio of two (finite) integers. So all quantities became commensurable, i.e., were integer multiples of a common unit. Although the Pythagorean theorem was probably known by the Chinese centuries earlier, it is considered to be their main mathematical fortress, and yet, it bears in it the irrationality of $\sqrt{2}$, contradicting their credo of commensurability.

And irrationals were everywhere. The spiral of Theodorus constructs all \sqrt{n} for integer n and the golden ratio was embedded in the pentagram, to which the Pythagoreans attributed a mystic value. Plato founded his school shortly after the discovery of the irrationals. One of its members, Eudoxus, actually succeeded in constructing geometric objects whose magnitude (length, area, volume) had a ratio m/n that was just larger or just smaller than the irrational number. This in fact precedes Dedekind cuts by 22 centuries.



scholars at Abbasid library

from *Maqamat* of al-Hariri, XIIth

Baghdad. It was a library and an intellectual center where the Greek scriptures were translated. Indian and Iranian scholars joined their knowledge and it are the Indians that gave us the decimal positional notation and, another major breakthrough: the zero. Where geometry was dominating

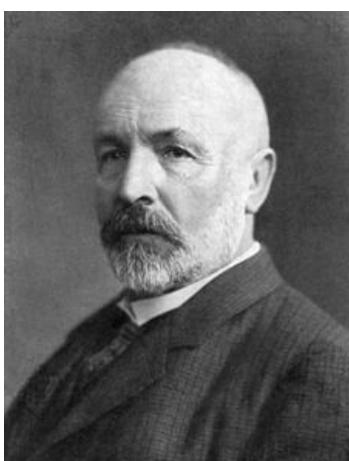
Eudoxus is in fact also the author of what we now refer to as the Archimedean axiom (the existence of infinitely small and infinitely large numbers). Archimedes used this idea to approximate a circle by regular n -gons for increasing n . Using inscribed and circumscribed approximations, converging to each other, we may recognize integral calculus here. Note also that this implies an implicit notion of infinity, it was never used explicitly by the Greek.

The Romans did not care too much about mathematics. It is a surprising for how long their impossible archaic number system has survived. After Plato's school was closed, the interesting scene moved to the East. It is at this point in history mainly thanks to the Islamic culture that the Greek mathematical heritage has been saved. Al-Mamoun, an Abbasidian caliph, founded a house of wisdom (the *Baid Al-Hikma*) in

mathematics for the Greek, here in the Abbasidian caliphate lies the birthplace of algebra. It was Mohammed Ibn Musa Al-Khwarizmi who described the Indian number system. Of this description only a Latin translation survived. Since he claimed to be original, the Indian system became known to us as the *arabic numerals*. Also our words ‘algebra’ is drawn from the title of his book *Hisab al-jabr wa'l muqabalah* and the latinization of his name led to our word ‘algorithm’. The Arabic word *sifir* for zero became our ‘cypher’. His fundamental conceptual contribution though is that he starts from the equations. The Arabs study polynomials and their roots, i.e., the algebraic irrationals. The Arabic texts, and hence also the Greek mathematics that had been translated into Arabic, were not translated into Latin but until the twelfth century. This contact between Christian and Arab cultures was not via the Middle East because the crusades did not allow a friendly exchange of scientific ideas, but it happened mainly in Spain where a relative coexistence was possible, until in 1492, when Baobdil, the last Arab king of Granada, was chased away and left for Africa.

That brings us back to Italy of the 15th century. Solving the cubic and quartic equation was a study already started by the Arabs, but although it is accepted that some Italians kept the formulas as a secret, it was Gerolamo Cardano, a professor in Bologna (yes, the very one whose name is associated with the Cardan joint in your car) who published the formulas in 1545. The formulas for solving the quadratic equation gave rise to square roots of negative numbers. Cardano did not know how to deal with this. His student Bombelli came close to an interpretation in 1579. However, society did not seem ripe for the complex numbers. The fact that they are not ordered gave rise to much confusion.

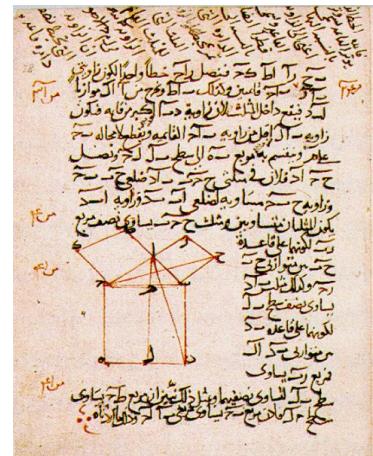
The translations of the Greek and Arabic texts that became generally available were quickly spread with the printing technique that had just been invented. The evolution of science and in particular mathematics now took off at an exponential rate. Newton and Leibniz invented calculus, which introduced the infinitesimals, not exactly numbers, but still.... Two numbers were considered equal not only when their difference is zero, but also when their difference is ‘incomparably small’. This tacitly introduced the concept of infinity. Besides the good old rationals and the algebraic numbers studied by the Italians before, now also the irrationals were entering the scene. If the complex roots of the cubic had perplexed Bombelli and Cardan, the periodicity of the complex exponential has even more confused the early workers of analysis. It was Euler in 1748 who introduced the complex exponential through its series expansion, which led to the famous relation $e^{i\pi} = -1$.



Georg Cantor (1845-1918)

It is difficult to bring the adventure of numbers at the same speed and detail as it was summarized till now. And we are only at about one third through the book. Also Godefroy needs to resort to higher mathematics since in the 19th century scholars were shaking the foundations of mathematics itself, and it all started with the real numbers that needed a precise axiomatic definition. Richard Dedekind introduced the reals as cuts (*Schnitte*): a real number is a partition of the rationals into those sets *A* and *B* such that every element of *A* is less than an element of *B*. Now, finally, the Archimedean axiom can be proved for the reals. Meanwhile, the algebraists introduced the quaternions (Hamilton) and later octonions (Graves and Cayley) as a further generalization of the complex numbers.

As Godefroy continues his story of the numbers, more mathematics enter the picture, and this book review is not the place to summarize them. Being half way in the book, we arrive in the 20th century, and this is probably

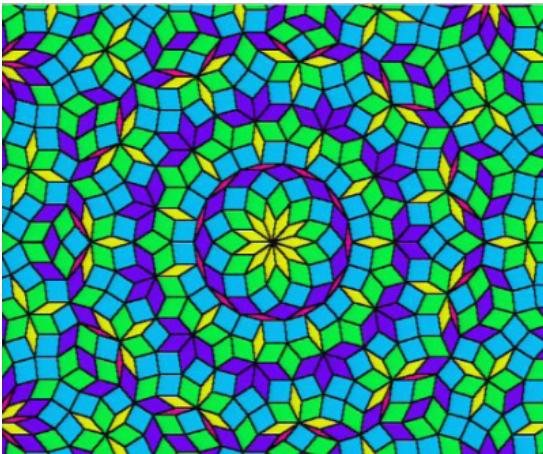


Nasrudeen Tusi’s record of Euclid’s proof of the Pythagorean Theorem

where the non-mathematician will loose track and where only the mathematicians will read on. And it is worth reading on because Godefroy succeeds in introducing the reader to all the major achievements in mathematics of the least century. Fermat's last theorem, Kummer's ideals, p -adic number systems,... A major step was set by Cantor who was far ahead of his time, and a separate chapter is devoted to him. He was so advanced that he was counteracted by many of his contemporaries which had to be silenced by the authority of Hilbert. Nevertheless this hostility led Cantor to depressions and he ended his days in a mental hospital. Cantor is the founder of set theory and Godefroy gives a step by step account of his diagonalization process, the introduction of transfinite numbers, the ordinals $\omega, \omega + 1, \omega + 2, \dots, \omega \cdot 2, \omega \cdot 2 + 1, \dots, \omega^2, \dots, \omega^3, \dots, \omega^\omega, \dots, \omega^{\omega^\omega}, \dots, \varepsilon_0, \dots$, and the cardinals: $0, 1, \dots, \aleph_0, \aleph_1, \dots$. He proved that for any set A , the power set 2^A of all subsets of A had more elements than the set A . This led to a contradiction since the set U of all sets would have a subset 2^U that is larger than U . Numbers had grown out of control.

So Hilbert in his famous speech at the ICM in Paris in 1900 set as one of his challenges to design a formal representation of calculus and set theory to avoid these inconsistencies. In an attempt to construct answers, Zermelo proved the well ordering of sets but thereby relied on the axiom of choice. This gives rise to the Banach-Tarski paradox: one may decompose a ball into finitely many pieces and recompose them to two balls with the same radius. The explanation is that the 'pieces' are abstract entities so that no volume can be assigned to them. Godefroy then moves on to the decision problem (conclude that a system is consistent or not), Gödel's theorem, non-Euclidean geometry, recursively enumerable sets and recursive sets, and the Robinson-Matiasevic theorem proved in 1970. It gives a negative answer to Hilbert's 10th problem asking for an automatic procedure in order to answer whether a diophantine equation has an integer solution. The author introduces the reader to model theory and nonstandard analysis.

In a concluding chapter entitled "And now?", Godefroy gives an update of current research near the end of the 20th century. Aperiodic tilings of the plane, fractals, dynamical systems, computer proofs, cryptography, the fast Fourier transform, Conway games and Conway numbers, quantum mechanics and Alain Connes's noncommutative geometry all get their couple of pages of fame here.



Aperiodic tiling in quasicrystal

mathematical layman will follow easily up to about halfway the book, but will leave the playground to mathematicians when the chapters dive into the foundations of set theory. For the mathematicians it should be clear from our summary that this is not a general history of mathematics. Godefroy clearly focusses on the numbers and the geometry, algebra, analysis, set theory and logic that is needed to pin down their properties.



Banach-Tarski paradox

A number of appendices and a glossary of terms and definitions contain some basic mathematical stuff to which is often referred in the text. There is however no index, which makes it difficult to look up names and events in the text. Another element that should be mentioned is that the author is regularly challenging the reader to check or elaborate on some formula. The solution of these is given at the end of the corresponding chapter.

Gilles Godefroy has a pleasant style of writing, showing a broad cultural background, although somewhat harmlessly biased towards French heritage. (Understandable given the fact that this is a translation of the French original.) As mentioned before, a mathe-

On the occasion of the 60th birthday
of our colleague and friend GILLES GODEFROY



ANALYSIS MEETING
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Anosov diffeomorphisms on infra-nilmanifolds · Jonas Deré

Introduction

Infra-nilmanifolds play a very important role in dynamical systems, especially when studying expanding maps or Anosov diffeomorphisms. Because of the algebraic nature of these manifolds, questions about self-maps can be translated into questions about endomorphisms of their fundamental groups. In this way, it was shown by M. Gromov that every expanding map on

a closed manifold is topologically conjugate to an affine infra-nilendomorphism (see [2]). Up to now it is unknown if a similar statement also holds for Anosov diffeomorphisms, although some partial results point in that direction, see e.g. [3] for the case of infra-nilmanifolds and [4] for the case of codimension one Anosov diffeomorphisms. This question of classifying all

Anosov diffeomorphisms on compact manifolds up to homeomorphism was already raised by S. Smale in [6]. This motivates my research about classifying infra-nilmanifolds admitting an Anosov diffeomorphism. My main result is a complete classification for infra-nilmanifolds modeled on a free nilpotent Lie group.

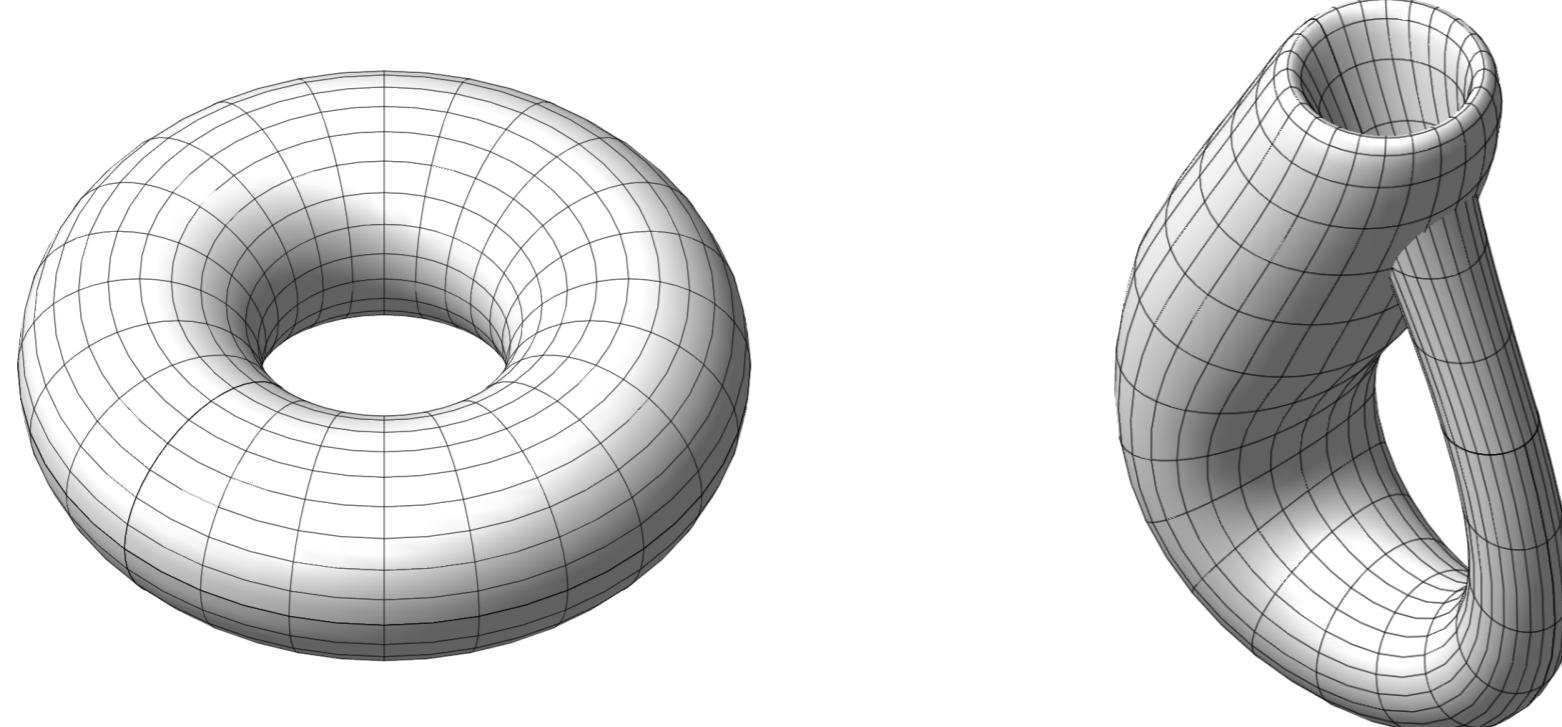
1. What is an infra-nilmanifold?

Let G be a connected and simply connected nilpotent Lie group. The affine group $\text{Aff}(G) = G \rtimes \text{Aut}(G)$ acts on G in the following natural way:

$$\forall \alpha = (g, \delta) \in \text{Aff}(G), \forall h \in G : {}^\alpha h = g\delta(h).$$

Let $F \subseteq \text{Aut}(G)$ be a finite group of automorphisms of G and let $\Gamma \subseteq G \rtimes F$ be a discrete and torsion-free subgroup such that the quotient $\Gamma \backslash G$ is compact. Such a group Γ is called an almost Bieberbach group (AB group) and the quotient space $\Gamma \backslash G$ is a closed manifold which we call an **infra-nilmanifold** modeled on the Lie group G . Let $p : \Gamma \rightarrow F$ be the natural projection on the second component, then we call $p(\Gamma)$ the holonomy group of $\Gamma \backslash G$. Infra-nilmanifolds are $K(\pi, 1)$ -spaces with π torsion-free, finitely generated and virtually nilpotent.

Example. If G is abelian, i.e. if G is the additive group \mathbb{R}^n for some n , then infra-nilmanifolds are exactly the closed flat manifolds, including all tori and the Klein bottle:



3. (Abelianized) Rational holonomy representation

Every AB group Γ with holonomy group F fits in a short exact sequence

$$1 \rightarrow N \rightarrow \Gamma \rightarrow F \rightarrow 1,$$

where $N = \Gamma \cap G$ is the subgroup of pure translations. By embedding the group N in its rational Mal'cev completion $N_{\mathbb{Q}}$, we find the following commutative diagram of groups

$$\begin{array}{ccccccc} 1 & \longrightarrow & N & \longrightarrow & \Gamma & \longrightarrow & F \longrightarrow 1 \\ & & \downarrow & & \downarrow & & \parallel \\ 1 & \longrightarrow & N_{\mathbb{Q}} & \longrightarrow & \Gamma_{\mathbb{Q}} & \longrightarrow & F \longrightarrow 1, \end{array}$$

where the last exact sequence splits. By fixing a splitting morphism $s : F \rightarrow \Gamma_{\mathbb{Q}}$, we define the rational holonomy representation $\varphi : F \rightarrow \text{Aut}(N_{\mathbb{Q}})$ by

$$\varphi(f)(n) = s(f)n s(f)^{-1}.$$

Since every automorphism of $N_{\mathbb{Q}}$ maps the commutator subgroup $[N_{\mathbb{Q}}, N_{\mathbb{Q}}]$ to itself, there also exists an abelianized rational holonomy representation

$$\bar{\varphi} : F \rightarrow \text{Aut}(N_{\mathbb{Q}}/[N_{\mathbb{Q}}, N_{\mathbb{Q}}]) \cong \text{GL}(n, \mathbb{Q})$$

for some $n \in \mathbb{N}$. This last representation allows us to use all methods of representation theory for finite groups. My main theorem extends an important result of Porteous (see [5]) for flat manifolds to infra-nilmanifolds modeled on a free nilpotent Lie group.

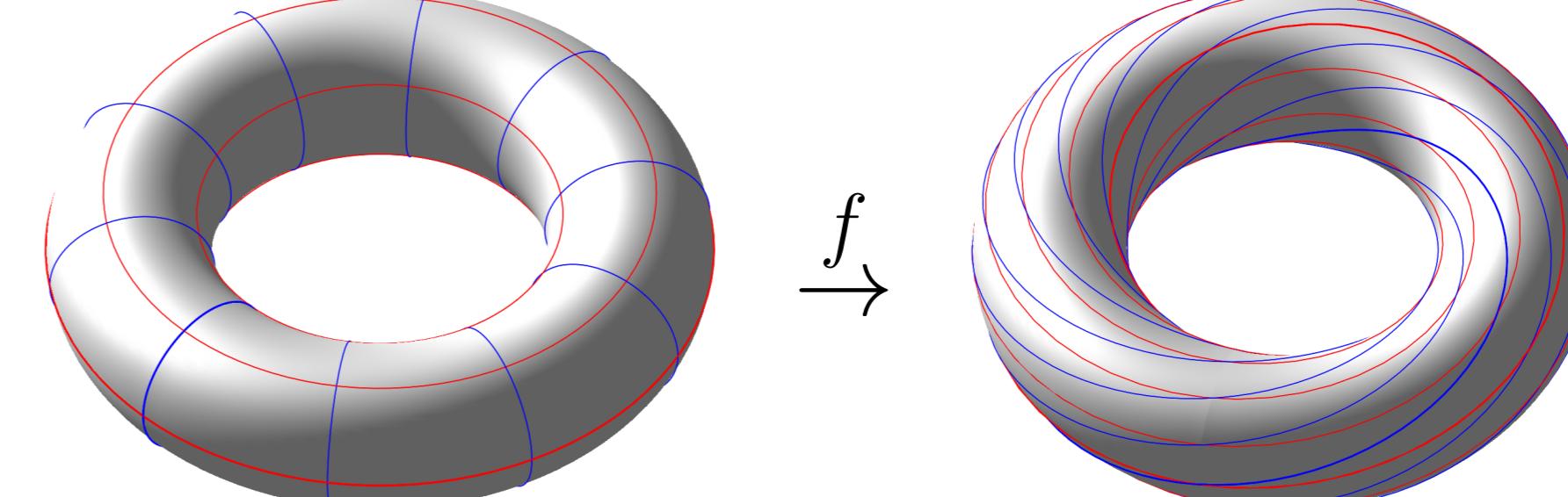
2. What is an Anosov diffeomorphism?

Let M be a closed manifold and $f : M \rightarrow M$ a C^1 -diffeomorphism. We call f an **Anosov diffeomorphism** if there exists a continuous splitting $TM = E^s \oplus E^u$ and a Riemannian metric on M such that:

- (i) The subbundles E^s and E^u are preserved under the map $Tf : TM \rightarrow TM$.
- (ii) There exists a constant $0 < \lambda < 1$ such that

$$\begin{aligned} \forall v \in E^u, \forall n \in \mathbb{N} : \|Tf^n(v)\| &\geq \lambda^{-n}\|v\|, \\ \forall v \in E^s, \forall n \in \mathbb{N} : \|Tf^n(v)\| &\leq \lambda^n\|v\|. \end{aligned}$$

Example. Consider the matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$, which induces a diffeomorphism f on the 2-torus $T^2 = \mathbb{Z}^2 \backslash \mathbb{R}^2$. This map f is also called Arnold's cat map:



More generally, every hyperbolic matrix $A \in \text{GL}(n, \mathbb{Z})$ induces an Anosov diffeomorphism on the n -torus $\mathbb{Z}^n \backslash \mathbb{R}^n$.

4. Classification for infra-nilmanifolds modeled on a free nilpotent Lie group

Theorem (Dekimpe K. and Deré J. ([1])). Let $\Gamma \backslash G$ be an infra-nilmanifold with G a free nilpotent Lie group of rank c and $\bar{\varphi} : F \rightarrow \text{GL}(n, \mathbb{Q})$ the abelianized rational holonomy representation. Then the following are equivalent:

$\Gamma \backslash G$ admits an Anosov diffeomorphism.

Every \mathbb{Q} -irreducible component of $\bar{\varphi}$ that occurs with multiplicity m , splits in more than $\frac{c}{m}$ components when seen as a representation over \mathbb{R} .

Sketch of the proof:

- (i) By working on a free nilpotent Lie group, the existence of an Anosov diffeomorphism is equivalent with the existence of a c -hyperbolic, integer-like matrix commuting with $\bar{\varphi}(F)$.
- (ii) The decomposition of $\bar{\varphi}$ into \mathbb{Q} -irreducible components allows us to assume that $\bar{\varphi} = \rho \oplus \dots \oplus \rho$ for some \mathbb{Q} -irreducible representation ρ of the holonomy group F .
- (iii) An extension field $E \supseteq \mathbb{Q}$ of minimal degree is determined such that ρ splits into \mathbb{C} -irreducible components, seen as a representation over E by extending the scalars.
- (iv) The construction of an explicit basis over which $\bar{\varphi}$ splits into \mathbb{C} -irreducible components allows to construct rational matrices commuting with $\bar{\varphi}$ starting from algebraic units in E .
- (v) The existence of c -hyperbolic, integer-like matrices commuting with every element $\bar{\varphi}(F)$ is then equivalent with the existence of sufficiently hyperbolic algebraic units in E and this depends only on the number of components of ρ over \mathbb{R} .

This proof also works for a more general class of nilpotent Lie groups, see [1] for the details.

5. Examples

1. For nilmanifolds (i.e. with holonomy $F = 1$) modeled on a free c -step nilpotent Lie group on n generators, the abelianized rational holonomy representation is trivial. So the \mathbb{Q} -irreducible and \mathbb{R} -irreducible components of $\bar{\varphi}$ are identical, implying that such a manifold allows an Anosov diffeomorphism if and only if $n > c$.

2. For the Klein bottle, the abelianized rational holonomy representation is given by

$$\bar{\varphi} : \mathbb{Z}_2 \rightarrow \text{GL}(2, \mathbb{Q}) : -1 \mapsto \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and thus it doesn't admit an Anosov diffeomorphism.

3. There exists an infra-nilmanifold with nontrivial holonomy modeled on the free 2-step nilpotent Lie group on 3 generators and the unique possibility for the abelianized rational holonomy representation is given by

$$\bar{\varphi} : \mathbb{Z}_2 \rightarrow \text{GL}(3, \mathbb{Q}) : -1 \mapsto \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

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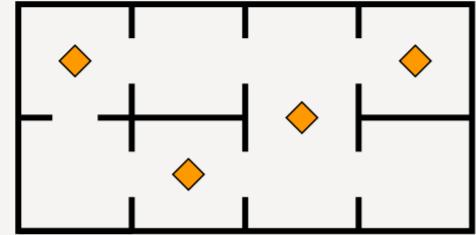
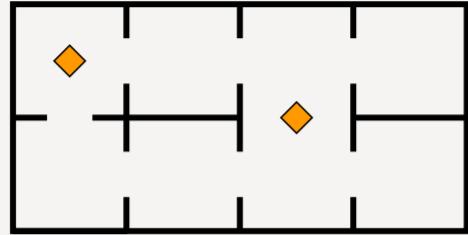
Linear formulation of identifying codes in graphs

VANDOMMÉ Elise University of Grenoble and University of Liège e.vandomme@ulg.ac.be
Joint work with GRAVIER Sylvain and PARREAU Aline

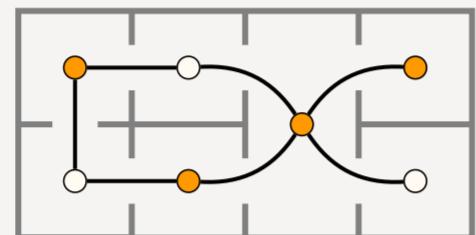
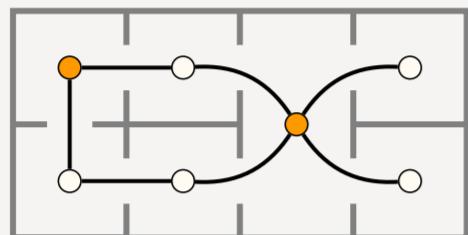
Fire in a building

Consider a sensor-placement in a building such that each sensor can detect a fire in its room and in its neighbouring rooms.

If there is a fire in one room, can we determine exactly where the fire is ?



Modelization with a graph : vertices : rooms,
edges : between two neighbouring rooms.



Identifying code of a graph : set C of vertices such that

- (**Domination**) each vertex must contain at least a vertex of C in its neighbourhood,
- (**Separation**) any pair of vertices can not have the same set of vertices of C in their neighbourhoods.

Linear formulation

We put weights $x_u \in \{0, 1\}$ on the vertices with the convention that for a set of vertices $C \subseteq V$, $x_u = 1 \Leftrightarrow u \in C$.

The set $C \subseteq V$ is an **identifying code** if for any $u, v \in V$ with $u \neq v$,

$$\sum_{w \in N[u]} x_w \geq 1 \quad \text{and} \quad \sum_{w \in N[u] \Delta N[v]} x_w \geq 1.$$

Domination

$$\sum_{w \in N[u] \Delta N[v]} x_w \geq 1.$$

Separation

Then $\gamma^{\text{ID}}(G) := \min\{\#C = \sum_{u \in V} x_u \mid C \text{ is an identifying code of } G\}$.

Finding an identifying code with minimum cardinality is **NP-hard** in general !

How to obtain a natural bound on $\gamma^{\text{ID}}(G)$?

Allow the weights x_u to be fractions in $[0, 1]$ in the above problem and denote by $\gamma_f^{\text{ID}}(G)$ the smallest possible $\sum_{u \in V} x_u$. Then

$$\gamma_f^{\text{ID}}(G) \leq \gamma^{\text{ID}}(G).$$

Case of transitive graphs

For a vertex-transitive graph G on n vertices, there exists an optimal solution to the fractional problem where $x_u = \frac{\gamma_f^{\text{ID}}(G)}{n}$ for any $u \in V$.

Let k be the degree of vertices and d the smallest cardinal of symmetric differences between two neighbourhoods. The domination and separation conditions imply that

$$\frac{n}{\gamma^{\text{ID}}(G)} \leq \min(k+1, d).$$

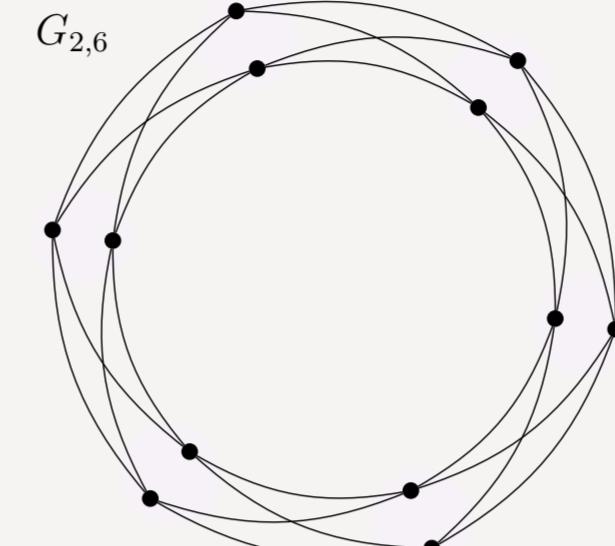
It is even an equality. Hence $\gamma_f^{\text{ID}}(G) = \max\left(\frac{n}{k+1}, \frac{n}{d}\right) \leq \gamma^{\text{ID}}(G)$.

NB : The bound $\frac{n}{k+1}$ is never reached. Indeed, Karpovsky, Chakrabarty and Levitin showed in 1998 that $\lceil \frac{2n}{k+2} \rceil \leq \gamma^{\text{ID}}(G)$.

When the separation condition prevails

How close is the bound $\gamma_f^{\text{ID}}(G) = \frac{n}{d}$ to $\gamma^{\text{ID}}(G)$?

A family of circulant graphs : $G_{m,p}$ with $m \geq 2, p > 4$



Number of vertices : $n = mp$

Degree of vertices : $k = 2m$

Smallest symmetric difference : $d = 2$

$$\gamma^{\text{ID}}(G) = p(m-1)$$

$$\frac{\gamma^{\text{ID}}(G)}{\gamma_f^{\text{ID}}(G)} = 2 - \frac{2}{m}$$

If $m = 2$, the bound is tight ! In contrast, the integer solution is nearly twice the fractional solution for big enough m . This corresponds to the worst case scenario with $d = 2$ since

$$\gamma^{\text{ID}}(G) \leq n = d\gamma_f^{\text{ID}}(G).$$

Cartesian product of complete graphs : $K_p \square K_p$

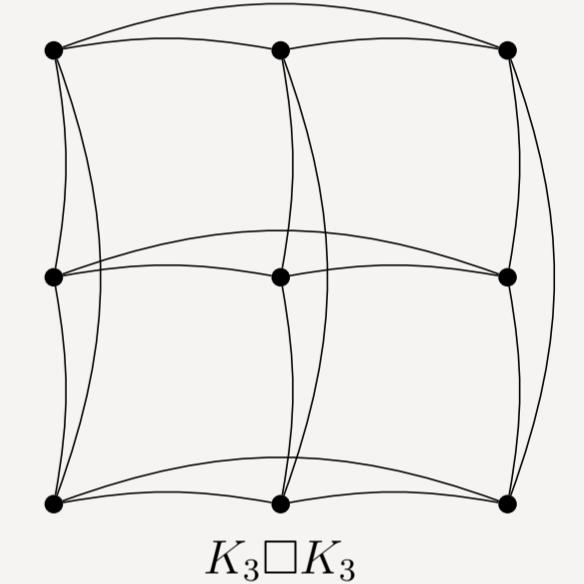
Number of vertices : $n = p^2$

Degree of vertices : $k = 2p-1$

Smallest symmetric difference : $d = 2p-2$

$$\gamma^{\text{ID}}(G) = \frac{3p}{2} \quad [\text{Gravier, Moncel, Semri (2008)}]$$

$$\frac{\gamma^{\text{ID}}(G)}{\gamma_f^{\text{ID}}(G)} = 3 - \frac{3}{p}$$



Q : Can we find examples with $d \neq 2$ corresponding to the “worst case”?

When the domination condition prevails

Generalized quadrangles $GQ(s, t)$

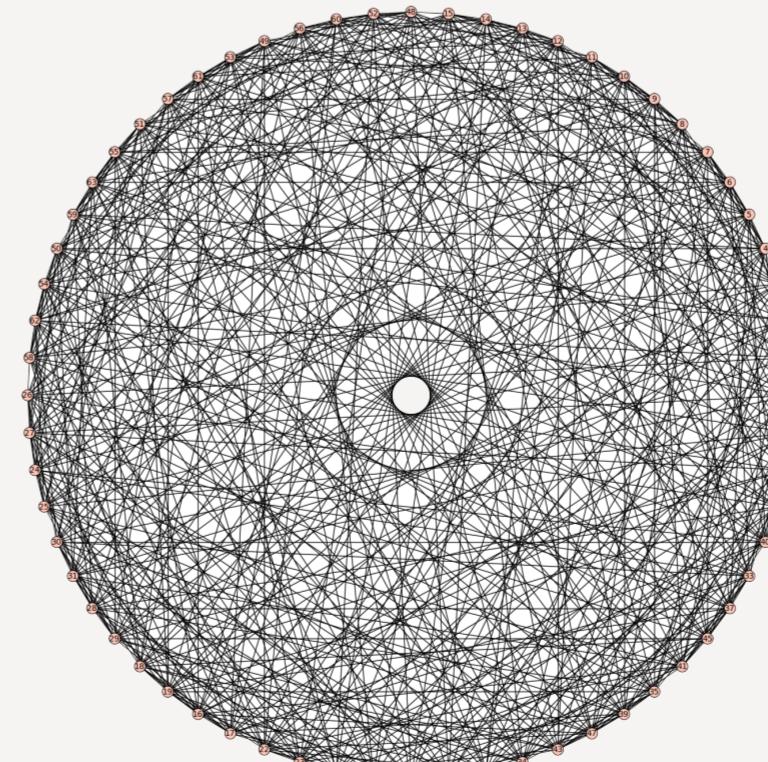
$GQ(s, t)$ is an incidence structure, i.e. a set of points and lines such that :

- there are $s+1$ points on each line,
- there are $t+1$ lines passing through a point,
- for a point P that does not belong to a line L , there is exactly one line passing through P and intersecting L .

Consider its **incidence graph** where points are vertices and there is an edge between two points if they belong to the same line.

For example, $K_p \square K_p$ corresponds to $GQ(p-1, 1)$.

The domination condition prevails in $GQ(s, t)$ iff $s > 1$ and $t > 1$.



$GQ(3, 5)$

Vertices : points of $\mathbb{F}_4^3 = \{0, 1, z, z^2\}^3$
Edges : between two points A and B if the direction (AB) belongs to

$$\{(1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 1), (1, z, z^2), (1, z^2, z)\}$$

Deg. of vertices : $k = 3 \cdot 6 = 18$

Smallest sym. diff. : $d = 26$

$$\gamma_f^{\text{ID}}(GQ(3, 5)) = \frac{64}{19} \approx 3,36 \text{ and } \gamma^{\text{ID}}(GQ(3, 5)) = 9$$

Q : Identifying code of $GQ(7, 9)$? Generalization to $GQ(2^\ell - 1, 2^\ell + 1)$?